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Investment Analysis

Tutorial

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МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РОССИЙСКОЙ ФЕДЕРАЦИИ

**Федеральное государственное автономное
образовательное учреждение высшего образования
«Национальный исследовательский Нижегородский государственный
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Институт экономики и предпринимательства

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Экономическая оценка инвестиций

Учебно-методическое пособие

*Рекомендовано методической комиссией института экономики и предпринимательства для иностранных студентов, обучающихся в ННГУ по направлению подготовки 38.03.01 «Экономика» (бакалавриат)
на английском языке*

Нижний Новгород

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В настоящем пособии изложены учебно-методические материалы по курсу «Экономическая оценка инвестиций» для иностранных студентов, обучающихся в ННГУ по направлению подготовки 38.03.01 «Экономика» (бакалавриат).

Учебно-методическое пособие предназначено для студентов института экономики и предпринимательства обучающихся по направлению подготовки 38.03.01 «Экономика

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Unit 1:   Investments and sources of its financing.

Investment - an asset or item that is purchased with the hope that it will generate income or appreciate in the future.

In an economic sense, an investment is the purchase of goods that are not consumed today but are used in the future to create wealth.

In finance, an investment is a monetary asset purchased with the idea that the asset will provide income in the future or appreciate and be sold at a higher price.

1.1.   Types of investment:
1.   Autonomous Investment

Investment which does not change with the changes in income level, is called as Autonomous or Government Investment.

Autonomous Investment remains constant irrespective of income level. Which means even if the income is low, the autonomous, Investment remains the same. It refers to the investment made on houses, roads, public buildings and other parts of Infrastructure. The Government normally makes such a type of investment.

2.   Induced Investment

Investment which changes with the changes in the income level, is called as Induced Investment.

Induced Investment is positively related to the income level. That is, at high levels of income entrepreneurs are induced to invest more and vice-versa. At a high level of income, Consumption expenditure increases this leads to an increase in investment of capital goods, in order to produce more consumer goods.

3.   Financial Investment

Investment made in buying financial instruments such as new shares, bonds, securities, etc. is considered as a Financial Investment.

However, the money used for purchasing existing financial instruments such as old bonds, old shares, etc., cannot be considered as financial investment. It is a mere transfer of a financial asset from one individual to another. In financial investment, money invested for buying of new shares and bonds as well as debentures have a positive impact on employment level, production and economic growth.

4.   Real Investment

Investment made in new plant and equipment, construction of public utilities like schools, roads and railways, etc., is considered as Real Investment.

Real investment in new machine tools, plant and equipments purchased, factory buildings, etc. increases employment, production and economic growth of the nation. Thus real investment has a direct impact on employment generation, economic growth, etc.

5.   Planned Investment

Investment made with a plan in several sectors of the economy with specific objectives is called as Planned or Intended Investment.

Planned Investment can also be called as Intended Investment because an investor while making investment make a concrete plan of his investment.

6.   Unplanned Investment

Investment done without any planning is called as an Unplanned or Unintended Investment.

In unplanned type of investment, investors make investment randomly without making any concrete plans. Hence it can also be called as Unintended Investment. Under this type of investment, the investor may not consider the specific objectives while making an investment decision.

7.   Gross Investment

Gross Investment means the total amount of money spent for creation of new capital assets like Plant and Machinery, Factory Building, etc.

It is the total expenditure made on new capital assets in a period.

8.   Net Investment

Net Investment is Gross Investment less (minus) Capital Consumption (Depreciation) during a period of time, usually a year.

It must be noted that a part of the investment is meant for depreciation of the capital asset or for replacing a worn-out capital asset. Hence it must be deducted to arrive at net investment

1.2.   Forms of business financing - Loans

Businesses often need to borrow money to finance business investment activities. Here are some of the types of loans a business might take out.

1. Basic Loans

A commercial loan is a debt-based funding arrangement that a business can set up with a financial institution. The proceeds of commercial loans may be used to fund large capital expenditures and/or operations that a business may otherwise be unable to afford. This type of loan is usually short-term in nature and is almost always backed with some sort of collateral. Commercial loans usually charge flexible rates of interest that are tied to the bank prime rate or else to the London Interbank Offered Rate (LIBOR). Many borrowers must file regular financial statements, usually at least annually. Lenders also usually require proper maintenance of the loan collateral property.

A term loan is a loan from a bank for a specific amount that has a specified repayment schedule and a floating interest rate. Term loans almost always mature between one and 10 years. Businesses use term loans for month-to-month operations or to purchase fixed assets such as production equipment.

An unsecured loan is issued and supported only by the borrower's creditworthiness, rather than by some sort of collateral. Generally, a borrower must have a high credit rating to receive an unsecured loan. Commercial paper is an

example of an unsecured loan. A secured loan is backed by collateral; if it is not repaid, the lender can seize the collateral and sell it to recover the funds it lent.

An acquisition loan helps a company purchase a specific asset that is determined before the loan is granted. Acquisition loans are sought when a company wants to complete an acquisition for an asset but does not have enough liquid capital to do so. The company may be able to get more favorable terms on an acquisition loan because the assets being purchased have a tangible value, as opposed to capital being used to fund daily operations or release a new product line. The acquisition loan is typically only available to be used for a short window of time and only for specific purposes. Once repaid, funds available through an acquisition loan cannot be re-borrowed as with a revolving line of credit at a bank.

2.   *Revolving Credit*

Revolving credit is another way businesses can borrow money, but the structure is a bit different than an ordinary loan.

A line of credit establishes a maximum loan balance that the bank will permit the borrower to maintain. The borrower can draw down on the line of credit at any time, as long as he or she does not exceed the maximum set in the agreement.

3.   *More Complex Loans*

A self-liquidating loan is a type of short or intermediate-term credit that is repaid with money generated by the assets purchased. The repayment schedule and maturity of a self-liquidating loan are designed to coincide with the timing of the assets' income generation. These loans are intended to finance purchases that will quickly and reliably generate cash.

An asset-conversion loan is a short-term loan that is typically repaid by converting an asset, usually inventory or receivables, into cash.

Another type of loan that can help a business meet its day to day needs is a cash flow loan. Reasons for needing a cash flow loan could be seasonal-demand changes, business expansion or changes in the business cycle

A working capital loan can also be used to finance everyday operations of a company. It is not used to buy long-term assets or investments, but rather to clear up accounts payable, pay wages and salaries, and so on.

A company can also pledge its accounts receivable (AR) as collateral for a loan. A non-notification loan is a type of full-recourse loan that is securitized by accounts receivable. Customers making accounts-receivable payments are not notified that their account/payment is being used as collateral for a loan. They continue making payments to the company that rendered services or made the original loan, and the company then uses those payments to repay their lender for financing obtained. If customers do not pay accounts receivable, the company is still liable for repaying the loan it obtained using the AR as security.

A bridge loan, also known as "interim financing," "gap financing" or a "swing loan," is a short-term loan that is used until a company secures permanent financing or removes an existing obligation. This type of financing allows the user to meet

current obligations by providing immediate cash flow. The loans are short term (up to one year) with relatively high interest rates and are backed by some form of collateral such as real estate or inventory.

This is not an exhaustive list of the types of loans available to businesses, but it gives a general idea of the different options available. Businesses should shop around at different institutions to determine which lender offers the best terms for the loan.

1.3. Forms of business financing - Alternatives to Loans

When conventional credit markets get tight, individuals and businesses are pushed to seek alternative lenders to obtain financing. Some of these alternative financing sources have been around for a long time, but the 2007-2008 credit crunch spawned some new potential financing sources for business owners and individuals, as well as some new ways to access them. Here are seven unconventional ways businesses can borrow money and the benefits, dangers and drawbacks of each.

1. Factoring

Factoring (also known as accounts receivable financing) is one of the oldest methods of in-house financing. Simply put, factoring is when a business sells its accounts receivable to a financial institution or "factor." The factor will advance funds on a portion of the receivables, usually 75-80% of their face value. The remaining 20-25% is known as the "reserve" and is initially held by the factor. The amount of the reserve will vary with the quality of the receivables and the historical average of the payers. Historically late payers will increase the amount of the required reserve.

2. Hedge-Fund Lenders

According to an August 2008 *Businessweek* article, hedge fund lenders are being referred to as "the new corporate ATMs." Hedge funds will often loan money into higher risk businesses, such as asset or technology-concept backed companies; the size of the loan will depend on the quality of the pitch made by the borrower. The decision to lend is usually made after some due diligence but with greater flexibility than that experienced with conventional lenders.

3. Peer-to-Peer Lenders

Peer-to-peer lenders may include family, friends and even strangers who are interested in your success. This can be a formal or informal arrangement. The benefits of this type of loan are quick access to cash and flexibility in the repayment requirements. This financing source may also have a downside: non-business issues and non-financial paybacks can get intertwined with the lending situation. Loans from family and friends may come with expectations of employment or free or discounted products from you or your business.

4. Customer Lenders

Borrowing from business customers started in the early 2000s with community supported agricultural loans (CSAs). In CSAs, farmers' customers loaned money

prior to the planting season and took payment in harvested product at discounted prices.

    5.   Credit Card Lenders

Often used by owners to start a business, financing from credit cards has the benefit of easy and early access to cash if your credit history is good.

    6.   Convertible Debt Instruments

Convertible debt instruments are essentially asset-backed loans that can require the business owner to give up some future equity in the business if the lender wishes to convert the debt to an equity position in the company.

Unit 2:   Investment projects.

2.1.   Definition of an investment project

A project is a proposal for an investment to create, expand and/or develop certain facilities in order to increase the production of goods and/or services in a community during a certain period of time Furthermore, for evaluation purposes, a project is a unit of investment which can be distinguished technically, commercially and economically from other investments

The construction of a new warehouse may not qualify as a project because even though it could be distinguished technically from the remainder of the factory, its functions are so closely interrelated with already existing parts of the plant that its commercial and social impact cannot reasonably be separated. On the other hand, the replacement of a fleet of delivery lorries by a railway siding with associated loading equipment may be a project because savings in transport costs connected with the measure could be made the object of separate commercial and economic appraisal.

2.2.   Project preparation and evaluation

Project development is an integrated process carried out in several consecutive phases which may be condensed into three stages: *project preparation, its evaluation and implementation.*

It is extremely important to point out that all three are closely interrelated and that the ultimate success of an investment decision depends equally on each of them.

Project preparation itself consists of a series of interdependent measures with the aim of translating an idea into an operating project This is done in different stages:

- Identification,
- Preliminary selection,
- Formulation.

Industrial project development starts with the identification of the project idea, a notion of possibility/desire to produce specific product(s) or to utilize specific resources. Project ideas may arise from studies of the product-consumption pattern of the country, market studies, surveys of existing industrial establishments, import schedules, internal resources, geological surveys, industrial linkages, sectoral and industry analyses, development plans, export possibilities, experience of other countries, increasing demand for manufactured inputs for different sectors, studies of technology and development literature etc. All ideas for projects are valuable and may prove to be the beginning of development

The identification of a project idea is followed by a preliminary selection stage. The objective at this stage is to decide whether a project idea should be studied in detail and what the scope should be of further studies. The findings at this stage are embodied in a pre-feasibility study (opportunity study)

The pre-feasibility study is carried out by an investor himself or by an investment promoter, e.g. a ministry or development agency. It is prepared on the basis of data that are available in published form or that can be easily collected or worked out.

Once it is proved that a project idea deserves detailed study, an investor should be found who would be interested in following it up (should the promoter not be identical with the investor). If the pre-feasibility study indicates that the proposed project appears to be a promising one, the decision may be taken to proceed further with the formulation of the project.

The function of the <u>formulation stage</u> is to study from the technical, economic, financial and managerial aspects all the alternative ways of accomplishing the objectives of the project idea, and to present the findings and supporting data in a systematic and logical order. This is done through partial (technical, management etc) or complete techno-economic feasibility studies.

The complete feasibility study is the final document in the formulation of a project proposal. On the basis of this study a decision to implement and finance the project will be taken

The feasibility study should contain all technical and economic data that are essential for the overall economic and social evaluation of a project. The feasibility study should be so self-contained that on the one hand the evaluator cannot complain of the lack of data or imperfect analysis and, on the other, the decision maker cannot find anything hidden or missing. Accumulation and presentation of all technical and economic facts in a true and complete picture should be the mam objective of this study.

The complete feasibility study is carried out by a consulting engineering firm, by a foreign supplier of equipment or by a potential investor who has the technical competence to accomplish this job.

The overall economic evaluation is a crucial exercise which is based on the project's feasibility report and precedes its implementation More specifically, the overall evaluation is a systematic procedure for weaving the technical and financial information about the project, with relevant data about its economic environment, together into one or a few criteria on the basis of which the project is recommended for selection, modification or rejection This procedure, however, does not mean that the evaluation of a project starts only when its preparation ends. Actually, project preparation and partial economic evaluation should be carried out simultaneously and are closely related. An overall economic evaluation is carried out only on the basis of data provided at the end of the formulation stage.

Interest in the technique of project evaluation has expanded significantly in recent years. Countries and businesses are seeking the articulation of, and refinements in, the criteria by which they would rationally sift projects competing for relatively limited resources.

What renders project evaluation an indispensable, though sometimes a rather elaborate, task is the existence of alternative economic opportunities for the commitment of resources, since the selection of a project would be considered rational only if that project is superior in some respect to others. Its superiority could be based on commercial profitability, i.e the net financial benefits accruing to the owners of the project.

The core of the evaluation process is somewhat similar and consists of three steps

a) Firstly, the identification of the quantity, quality and timing of physical inputs and outputs,

b) Secondly, the attachment of appropriate prices to the inputs and outputs in order to compute the respective values of costs and benefits,

c) Thirdly, the commeasurement of costs and benefits of the project in such a way that facilitates its comparison with alternative projects

The entire process leading up to a project's implementation in reality is seldom a clear-cut, step-by-step procedure as described above In practice, evaluation may reveal that certain aspects of a project have to be re-prepared. Similarly, project implementation may encounter unforeseen difficulties which require both redesigning certain project elements and evaluating the impact of this redesigning on the project's overall merits

Unit 3: Basic information needed for project evaluation

Project evaluation is largely a quantitative exercise. A solid data base, therefore, is required to form a judgment on a project. In collecting these data the evaluator normally has to rely on information supplied by the investor and his consultants. The purpose of the various stages of project preparation is in fact to establish the magnitudes, both in physical and monetary terms, surrounding the construction and operation of an investment project. Ultimately, these magnitudes are brought together in a techno-economic feasibility study which is the starting point for an overall project evaluation More often than not, however, it is up to the evaluator to organize the data in a manner to suit the appraisal methods that he intends to apply

Financial reporting is the method a firm uses to convey its financial performance to the market, its investors, and other stakeholders. The objective of financial reporting is to provide information on the changes in a firm's performance and financial position that can be used to make financial and operating decisions. In addition to being a management aid, this information is used by analysts to forecast the firm's ability to produce future earnings and as a means to assess the firm's intrinsic value. Other stakeholders, such as creditors, will use financial statements as a way to evaluate the company's economic and competitive strength.

Knowing how to work with the numbers in a company's financial statements is an essential skill. The meaningful interpretation and analysis of balance sheets, income statements and cash flow statements to discern a company's investment qualities is the basis for smart investment choices.

The Balance Sheet

The balance sheet provides information on what the company owns (its assets), what it owes (its liabilities) and the value of the business to its stockholders (the shareholders' equity) as of a specific date. It's called a balance sheet because the two sides balance out. This makes sense: a company has to pay for all the things it has (assets) by either borrowing money (liabilities) or getting it from shareholders (shareholders' equity).

The Income Statement

The income statement measures a company's financial performance over a specific accounting period. Financial performance is assessed by giving a summary of how the business incurs its revenues and expenses through both operating and non-operating activities. It also shows the net profit or loss incurred over a specific accounting period, typically over a fiscal quarter or year. The income statement is also known as the "profit and loss statement" or "statement of revenue and expense."

The Statement of Cash Flow

The statement of cash flow reports the impact of a firm's operating, investing and financial activities on cash flows over an accounting period.

The statement of cash flows is segregated into three sections:

- Operating activities
- Investing activities
- Financing activities

1. Cash Flow from Operating Activities (CFO)

CFO is cash flow that arises from normal operations such as revenues and cash operating expenses net of taxes. This includes:

Cash inflow (+)	Cash outflow (-)
Revenue from sale of goods and services	Payments to suppliers
Interest (from debt instruments of other entities)	Payments to employees
Dividends (from equities of other entities)	Payments to government
	Payments to lenders
	Payments for other expenses

2. Cash Flow from Investing Activities (CFI)

CFI is cash flow that arises from investment activities such as the acquisition or disposition of current and fixed assets. This includes:

Cash inflow (+)	Cash outflow (-)
Sale of property, plant and equipment	Purchase of property, plant and equipment
Sale of debt or equity securities (other entities)	Purchase of debt or equity securities (other entities)
Collection of principal on loans to other entities	Lending to other entities

3. Cash flow from financing activities (CFF)

CFF is cash flow that arises from raising (or decreasing) cash through the issuance (or retraction) of additional shares, or through short-term or long-term debt for the company's operations. This includes:

Cash inflow (+)	Cash outflow (-)
Sale of equity securities	Dividends to shareholders
Issuance of debt securities	Redemption of long-term debt
	Redemption of capital stock

Free Cash Flow

By establishing how much cash a company has after paying its bills for ongoing activities and growth, FCF is a measure that aims to cut through the arbitrariness and "guesstimations" involved in reported earnings. Regardless of whether a cash outlay is counted as an expense in the calculation of income or turned into an asset on the balance sheet, free cash flow tracks the money.

To calculate FCF, make a beeline for the company's cash flow statement and balance sheet. There you will find the item cash flow from operations (also referred to as "operating cash"). From this number, subtract estimated capital expenditure required for current operations:

Cash Flow From Operations - Capital Expenditure= Free Cash Flow

To do it another way, grab the income statement and balance sheet. Start with net income and add back charges for depreciation and amortization. Make an additional adjustment for changes in working capital, which is done by subtracting current liabilities from current assets. Then subtract capital expenditure (or spending on plants and equipment):

Net income + Depreciation/Amortization - Change in Working Capital - - Capital Expenditure = Free Cash Flow

It might seem odd to add back depreciation/amortization since it accounts for capital spending. The reasoning behind the adjustment is that free cash flow is meant to measure money being spent right now, not transactions that happened in the past. This makes FCF a useful instrument for identifying growing companies with high up-front costs, which may eat into earnings now but have the potential to pay off later.

Growing free cash flows are frequently a prelude to increased earnings. Companies that experience surging FCF - due to revenue growth, efficiency improvements, cost reductions, share buy backs, dividend distributions or debt elimination - can reward investors tomorrow. That is why many in the investment community cherish FCF as a measure of value. When a firm's share price is low and free cash flow is on the rise, the odds are good that earnings and share value will soon be heading up.

By contrast, shrinking FCF signals trouble ahead. In the absence of decent free cash flow, companies are unable to sustain earnings growth. An insufficient FCF for earnings growth can force a company to boost its debt levels. Even worse, a company without enough FCF may not have the liquidity to stay in business.

Unit 4:   Time Value Of Money

4.1.   Introduction

The idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity is called the time value of money. This core principle of finance holds that, provided money can earn interest, any amount of money is worth more the sooner it is received. Thus, at the most basic level, the time value of money demonstrates that, all things being equal, it is better to have money now rather than later.
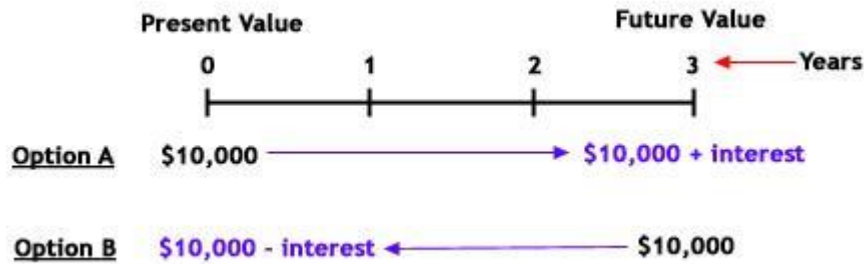
There are two ways to calculate Future Value (FV):
1) For an asset with simple annual interest:
$$FV = PV \times (1 + r \times n)$$
    Where

FV – Future Value

PV – Present Value (original investment)

r – Interest rate per period

n – Number of Period

    $1000 invested for five years with simple annual interest of 10% would have a future value of $1,500.00.

2) For an asset with interest compounded annually:
$$FV = PV \times (1 + r)^n$$
$1000 invested for five years at 10%, compounded annually has a future value of $1,610.51.

Note in the example below that when you increase the frequency of compounding, you also increase the future value of your investment.

PV = $10,000

n = 10 years

r = 9%

Example 1- If interest is compounded annually, the future value (FV) is $23,674.

$$FV = \$10,000(1 + 0.09)^{10} = \$23,674$$

Example 2 - If interest is compounded monthly, the future value (FV) is $24,514.

$$FV = \$10,000(1 + 0.09/12)^{120} = \$24,514$$

Present value, also called "discounted value," is the current worth of a future sum of money or stream of cash flow given a specified rate of return. Future cash flows are discounted at the discount rate; the higher the discount rate, the lower the present value of the future cash flows. Determining the appropriate discount rate is the key to properly valuing future cash flows, whether they are earnings or obligations.

The above future value equation can be manipulating to:

$$PV = \frac{FV}{(1+r)^n} = FV \times (1+r)^{-n}$$

Example You could receive either $15,000 today or $18,000 in four years. Which would you choose if interest rates are currently 4%. ?

$$PV = FV \times (1+r)^{-n} = \$18000 \times (1+0.04)^{-4} = \$15386.48$$

From the above calculation we now know our choice is between receiving $15,000 or $15,386.48 today. Of course we should choose to postpone payment for four years

These calculations demonstrate that time literally is money - the value of the money you have now is not the same as it will be in the future and vice versa. It is important to know how to calculate the time value of money so that you can distinguish between the worth of investments that offer you returns at different times.

4.2. Discounted Cash Flow Valuation

Discounted cash flow (DCF) is a valuation method used to estimate the attractiveness of an investment opportunity. DCF analysis uses future free cash flow projections and discounts them to arrive at a present value, which is then used to evaluate the potential for investment. If the value arrived at through DCF analysis is higher than the current cost of the investment, the opportunity may be a good one.

The formula for calculating DCF is usually given something like this:

$$PV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_{term}}{(r-g)} \times \frac{1}{(1+r)^{n-1}}$$

Where:

PV = present value

CF_i = cash flow in year i

r = discount rate

CF_{term} = the terminal year cash flow

g = growth rate assumption in perpetuity beyond terminal year
n = the number of periods in the valuation model including the terminal year

4.2.1. The Future Value And Present Value of Annuities

Annuities are essentially series of fixed payments required from you or paid to you at a specified frequency over the course of a fixed period of time. The most common payment frequencies are yearly (once a year), semi-annually (twice a year), quarterly (four times a year) and monthly (once a month). There are two basic types of annuities: ordinary annuities and annuities due.
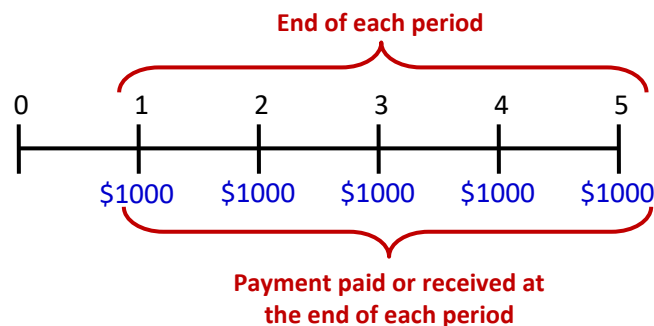
- Ordinary Annuity: Payments are required at the end of each period. For example, straight bonds usually pay coupon payments at the end of every six months until the bond's maturity date.
- Annuity Due: Payments are required at the beginning of each period. Rent is an example of annuity due. You are usually required to pay rent when you first move in at the beginning of the month and then on the first of each month thereafter.

Since the present and future value calculations for ordinary annuities and annuities due are slightly different, we will first discuss the present and future value calculation for ordinary annuities.

Calculating the Future Value of an Ordinary Annuity

If you know how much you can invest per period for a certain time period, the future value of an ordinary annuity formula is useful for finding out how much you would have in the future by investing at your given interest rate. If you are making payments on a loan, the future value is useful for determining the total cost of the loan.

Let's now run through Example 1. Consider the following annuity cash flow schedule:

End of each period

0   1   2   3   4   5

$1000 $1000 $1000 $1000 $1000

Payment paid or received at the end of each period

In order to calculate the future value of the annuity, we have to calculate the future value of each cash flow. Let's assume that you are receiving $1,000 every year for the next five years, and you invested each payment at 5%. The following diagram shows how much you would have at the end of the five-year period:

$$= \$1000*(1.05)^0 = \$1000.00$$
$$= \$1000*(1.05)^1 = \$1050.00$$
$$= \$1000*(1.05)^2 = \$1102.50$$
$$= \$1000*(1.05)^3 = \$1157.63$$
$$= \$1000*(1.05)^4 = \$1215.51$$

Future Value of an Ordinary Annuity = $5525.64
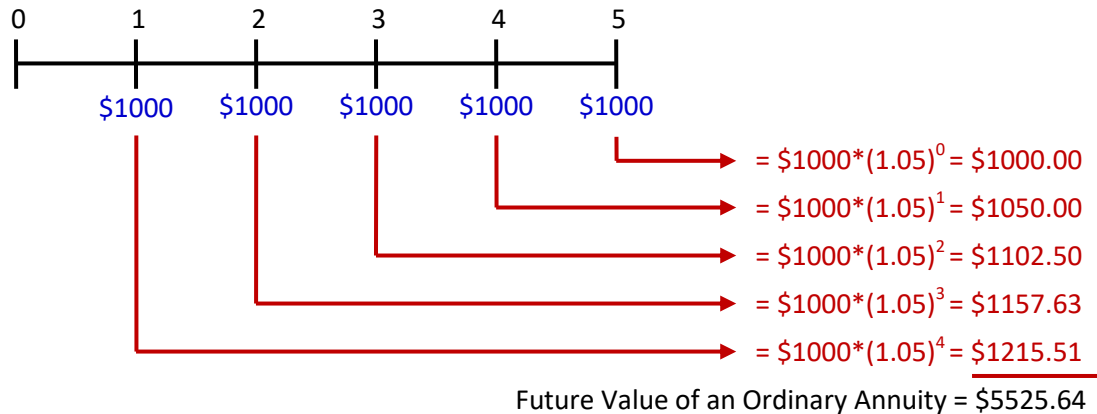
Since we have to add the future value of each payment, you may have noticed that, if you have an ordinary annuity with many cash flows, it would take a long time to calculate all the future values and then add them together. Fortunately, there's a formula that serves as a short cut for finding the accumulated value of all cash flows received from an ordinary annuity:

$$FV_{Ordinary\ Annuity} = CF \times \left[\frac{(1+r)^n - 1}{r} \right]$$

CF = Cash flow per period (Payment)
r = interest rate
n = number of payments
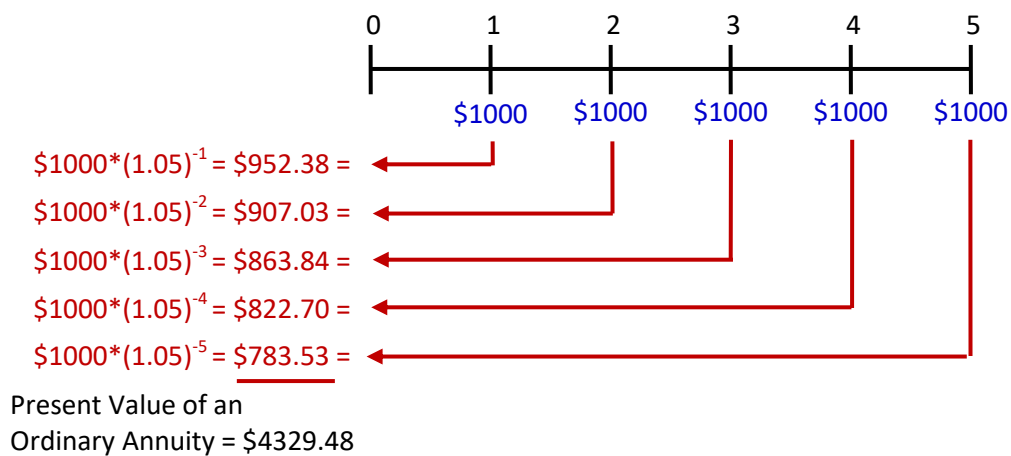If we were to use the above formula for Example 1 above, this is the result:

$$FV_{Ordinary\ Annuity} = CF \times \left[\frac{(1+r)^n - 1}{r} \right] = \$1000 \times \left[\frac{(1+0.05)^5 - 1}{0.05} \right] = \$5525.63$$

Note that the one cent difference between $5,525.64 and $5,525.63 is due to a rounding error in the first calculation. Each of the values of the first calculation must be rounded to the nearest penny - the more you have to round numbers in a calculation the more likely rounding errors will occur. So, the above formula not only provides a short-cut to finding the future value (FV) of an ordinary annuity but also gives a more accurate result.

Calculating the Present Value of an Ordinary Annuity
If you would like to determine today's value of a series of future payments, you need to use the formula that calculates the present value (PV) of an ordinary annuity. This is the formula you would use as part of a bond pricing calculation. The PV of ordinary annuity calculates the present value of the coupon payments that you will receive in the future.

For Example 2, we'll use the same annuity cash flow schedule as we did in Example 1. To obtain the total discounted value, we need to take the present value of each future payment and, as we did in Example 1, add the cash flows together.

$1000*(1.05)^{-1} = \$952.38 =$

$1000*(1.05)^{-2} = \$907.03 =$

$1000*(1.05)^{-3} = \$863.84 =$

$1000*(1.05)^{-4} = \$822.70 =$

$1000*(1.05)^{-5} = \$783.53 =$

Present Value of an
Ordinary Annuity = $4329.48

Again, calculating and adding all these values will take a considerable amount of time, especially if we expect many future payments. As such, there is a mathematical shortcut we can use for the PV of an ordinary annuity.

$$PV_{Ordinary\ Annuity} = CF \times \left[\frac{1-(1+r)^{-n}}{r}\right]$$

CF = Cash flow per period (Payment)
r = interest rate
n = number of payments

The formula provides us with the PV in a few easy steps. Here is the calculation of the annuity represented in the diagram for Example 2:

$$PV_{Ordinary\ Annuity} = CF \times \left[\frac{1-(1+r)^{-n}}{r}\right] = \$1000 \times \left[\frac{1-(1+0.05)^{-5}}{0.05}\right] = \$4329.48$$

Calculating the Future Value of an Annuity Due
When you are receiving or paying cash flows for an annuity due, your cash flow schedule would appear as follows:

20
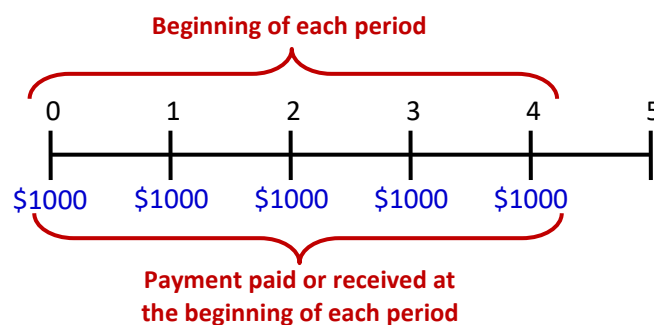
Since each payment in the series is made one period sooner, we need to discount the formula one period back. A slight modification to the FV-of-an-ordinary-annuity formula accounts for payments occurring at the beginning of each period. In Example 3, let's illustrate why this modification is needed when each $1,000 payment is made at the beginning of the period rather than the end (assuming the interest rate is still 5%):
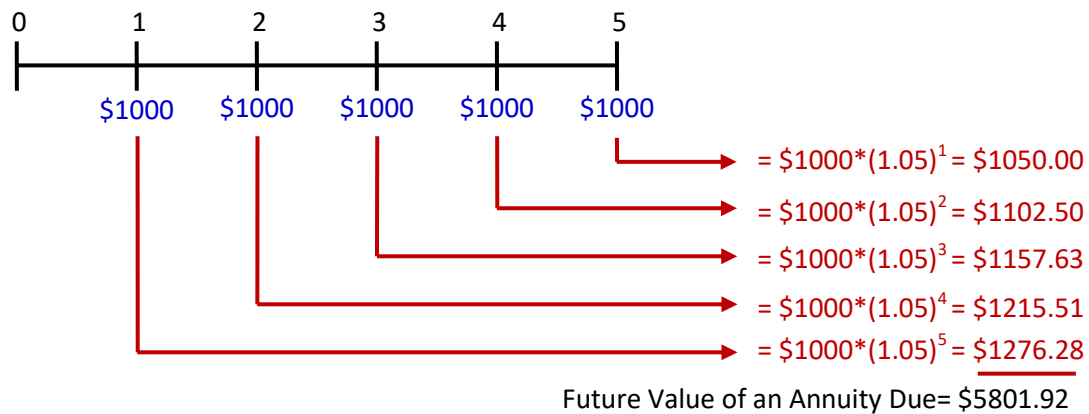

| 0 | 1 | 2 | 3 | 4 | 5 |

$1000 $1000 $1000 $1000 $1000

$= \$1000*(1.05)^1 = \$1050.00$
$= \$1000*(1.05)^2 = \$1102.50$
$= \$1000*(1.05)^3 = \$1157.63$
$= \$1000*(1.05)^4 = \$1215.51$
$= \$1000*(1.05)^5 = \$1276.28$

Future Value of an Annuity Due= $5801.92

Notice that when payments are made at the beginning of the period, each amount is held for longer at the end of the period. For example, if the $1,000 was invested on January 1st rather than December 31st of each year, the last payment before we value our investment at the end of five years (on December 31st) would have been made a year prior (January 1st) rather than the same day on which it is valued. The future value of annuity formula would then read:

$$FV_{Annuity\ Due} = CF \times \left[\frac{(1+r)^n - 1}{r} \right] \times (1+r)$$

Therefore,

$$FV_{Annuity\ Due} = \$1000 \times \left[\frac{(1+0.05)^5 - 1}{0.05} \right] \times (1+0.05) = 5801.91$$

Calculating the Present Value of an Annuity Due

For the present value of an annuity due formula, we need to discount the formula one period forward as the payments are held for a lesser amount of time. When calculating the present value, we assume that the first payment was made today.

We could use this formula for calculating the present value of your future rent payments as specified in a lease you sign with your landlord. Let's say for Example 4 that you make your first rent payment at the beginning of the month and are evaluating the present value of your five-month lease on that same day. Your present value calculation would work as follows:

Of course, we can use a formula shortcut to calculate the present value of an annuity due:

$$PV_{Annuity\ Due} = CF \times \left[\frac{1-(1+r)^{-n}}{r} \right] \times (1+r)$$
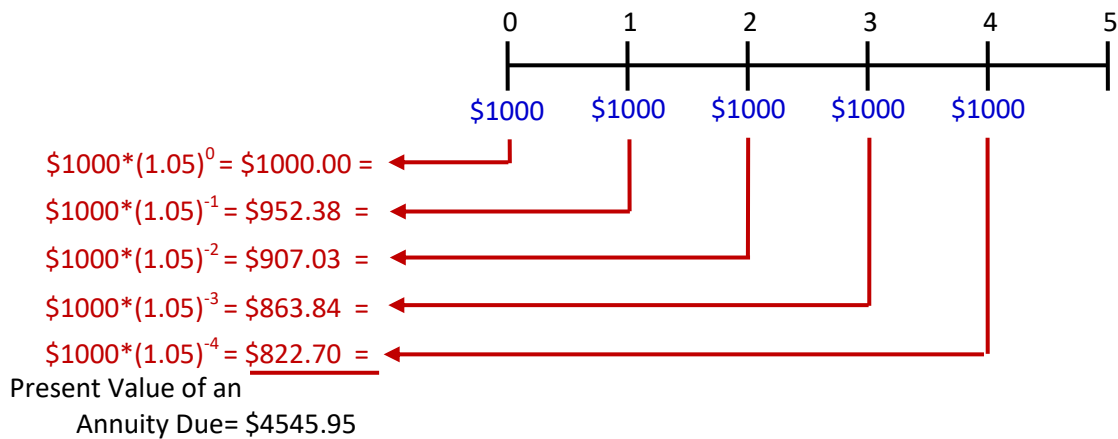
Therefore,

$$PV_{Annuity\ Due} = \$1000 \times \left[\frac{1-(1+0.05)^{-5}}{0.05} \right] \times (1+0.05) = \$4545.95$$

Recall that the present value of an ordinary annuity returned a value of $4,329.48. The present value of an ordinary annuity is less than that of an annuity due because the further back we discount a future payment, the lower its present value as each payment or cash flow in ordinary annuity occurs one period further into future.

Now you can see how annuity affects how you calculate the present and future value of any amount of money. Remember that the payment frequencies (or number of payments) and the time at which these payments are made (whether at the beginning or end of each payment period) are all variables you need to account for in your calculations.

4.2.2. Perpetuities

A perpetuity is a constant stream of identical cash flows with no end. The formula for determining the present value of a perpetuity is as follows:

$$PV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots = \frac{CF}{r}$$

A delayed perpetuity is perpetual stream of cash flows that starts at a predetermined date in the future. For example, preferred fixed dividend paying shares are often valued using a perpetuity formula. If the dividends are going to originate (start) five years from now, rather than next year, the stream of cash flows would be considered a delayed perpetuity.

Although it may seem a bit illogical, an infinite series of cash flows can have a finite present value. Because of the time value of money, each payment is only a fraction of the last.

The net present value (NPV) of a delayed perpetuity is less than a comparable ordinary perpetuity because, based on time value of money principles, the payments have to be discounted to account for the delay. Retirement products are often structured as delayed perpetuities.

Examples of Perpetuities

The perpetuity is not as abstract a concept as you may think. The British-issued bonds, called consols, are a great example of a perpetuity. By purchasing a consol from the British government, the bondholder is entitled to receive annual interest payments forever.

Another example is a type of government bond called an undated issue that has no maturity date and pays interest in perpetuity. While the government can redeem an undated issue if it so chooses, since most existing undated issues have very low coupons, there is little or no incentive for redemption. Undated issues are treated as equity for all practical purposes due to their perpetual nature, but are also known as perpetual bonds.

Perhaps the best-known undated issues are the U.K. government's undated bonds or gilts, of which there are eight issues in existence, some of which date back to the 19th century. The largest of these issues presently is the War Loan, with an issue size of £1.9 billion and a coupon rate of 3.5% that was issued in the early 20th century.

Perpetuities and the Dividend Discount Model

The concept of a perpetuity is used often in financial theory, particularly with the dividend discount model (DDM). Unfortunately, the theory is the easy part. The model requires a number of assumptions about a company's dividend payments, growth patterns and future interest rates. Difficulties spring up in the search for sensible numbers to fold into the equation. Here we'll examine this model and show you how to calculate it.

The basic idea is that any stock is ultimately worth no more than what it will provide investors in current and future dividends. Financial theory says that the value of a stock is worth all of the future cash flows expected to be generated by the firm, discounted by an appropriate risk-adjusted rate. According to the DDM, dividends are the cash flows that are returned to the shareholder.

To value a company using the DDM, calculate the value of dividend payments that you think a stock will generate in the years ahead. Here is what the model says:

$$\textit{Zero Growth:} \qquad P_0 = \frac{Div}{r}$$

Where:
P= the price at time 0
Div = dividend payment

r= discount rate

For simplicity's sake, consider a company with a $1 annual dividend. If you figure the company will pay that dividend indefinitely, you must ask yourself what you are willing to pay for that company. Assume the expected return (or the required rate of return) is 5%. According to the dividend discount model, the company should be worth $20 ($1.00 / 0.05).

How do we get to the formula above? It's actually just an application of the formula for a perpetuity:

$$P_0 = \frac{Div_1}{(1+r)^1} + \frac{Div_2}{(1+r)^2} + \ldots = \frac{Div}{r}$$

The obvious shortcoming of the model above is that you'd expect most companies to grow over time. If you think this is the case, then the denominator equals the expected return less the dividend growth rate. This is known as the constant growth DDM or the Gordon model after its creator, Myron Gordon. Let's say you think the company's dividend will grow by 3% annually. The company's value should then be $1 / (0.05 - 0.03) = $50. Here is the formula for valuing a company with a constantly growing dividend, as well as the proof of the formula:

$$\textit{Costant Growth:} \qquad P_0 = \frac{Div}{r-g}$$

$$P_0 = \frac{Div_1}{(1+r)^1} + \frac{Div_2(1+g)}{(1+r)^2} + \frac{Div_2(1+g)^2}{(1+r)^3} + \ldots = \frac{Div}{r-g}$$

The classic dividend discount model works best when valuing a mature company that pays a hefty portion of its earnings as dividends, such as a utility company.

Unit 5:   Evaluation of an investment project

Different methods may be used to assess the investment profitability of a project:
- Average rate of return
- Pay-back period
- Net present value
- Internal rate of return
- Profitability index
- Discounted Payback Period

The first two methods, the average rate of return and the pay-back period, are usually referred to as the simple or static methods since they do not take into consideration the whole life span of the project but rely on one model period (most frequently one year) or at best on a few periods. Furthermore, their application is based on the project's annual data, meaning that all the inflows and outflows enter the analysis at their nominal non-discounted values as they appear at a given time during the project's life.

The other methods are called discounted or dynamic methods because they take into consideration the entire life of a project and the time value of money by discounting the future inflows and outflows to their present values.

Hence, the simple methods are somewhat less precise, but in some cases a simple analysis could be sufficient and the only possible alternative while in others it would be preferable to carry out comprehensive analysis using the net present value and the internal rate of return methods.

5.1.   Average rate of return (ARR)

Average rate of return, also called accounting rate of return, provides a quick estimate of a project's worth over its useful life. ARR is calculated by finding a capital investment's average operating profits before interest and taxes but after depreciation and amortization (also known as "EBIT") and dividing that number by the book value of the average amount invested. It can be expressed as the following:

$$ARR = \frac{Average\,profit}{Average\,investment}$$

The result is expressed as a percentage. In other words, ARR compares the amount invested to the profits earned over the course of a project's life. The higher the ARR, the better.

The major drawbacks of ARR are as follows:

1. It uses operating profit rather than cash flows. Some capital investments have high upkeep and maintenance costs, which bring down profit levels.

2. Unlike NPV and IRR, it does not account for the time value of money. By ignoring the time value of money, the capital investment under consideration will

appear to have a higher level of return than what will occur in reality. The capital investment may appear to be more lucrative than the alternatives, such as investing in the financial markets, when it is actually less lucrative.

Here is a simple example of an ARR calculation: A project requiring an average investment of $1,000,000 and generating an average annual profit of $150,000 would have an ARR of 15%.

While ARR is easy to calculate and can be used to gauge the results of other capital budgeting calculations, it is not the most accurate metric.

5.2. Payback Period (PB)

The payback rule, also called the payback period, is the length of time required to recover the cost of an investment.

The payback period is calculated as follows:

$$PB = \frac{Cost\ of\ project}{Annual\ Cash\ Inflows}$$

All other things being equal, the better investment is the one with the shorter payback period.

For example, if a project costs $100,000 and is expected to return $20,000 annually, the payback period will be $100,000 / $20,000, or five years.

Here is another example. If a capital budgeting project requires an initial cash outlay of $1 million, the PB reveals how many years are required to for the cash inflows to equate to the one million dollar outflow. A short PB period is preferred as it indicated that the project will "pay for itself" within a smaller time frame.

In the following example, the PB period would be three and one-third of a year, or three years and four months.

Investment	Inflows				
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
-1,000,000	300,000	300,000	300,000	300,000	300,000

Payback periods are typically used when liquidity presents a major concern. If a company only has a limited amount of funds, they might be able to only undertake one major project at a time. Therefore, management will heavily focus on recovering their initial investment in order to undertake subsequent projects. Another major advantage of using the PB is that it is easy to calculate once the cash flow forecasts have been established.

While the payback rule appears very straightforward, there are two significant problems with this method.

- It ignores the time value of money.
- It ignores any benefits that occur after the payback period and therefore does not measure profitability.

Because of these reasons, other methods of capital budgeting like net present value, internal rate of return or discounted cash flow are generally preferred.

Let's take a closer look at the two major drawbacks to using the PB metric to determine capital budgeting decisions. Firstly, the payback period does not account for time value of money. Simply calculating the PB provides a metric which places the same emphasis on payments received in year one and year two. Such an error violates one of the basic fundamental principles of finance. Luckily, this problem can easily be amended by implementing a discounted payback period model. Basically, the discounted PB period allows one to determine how long it take for the investment to be recovered on a discounted cash flow basis.

The second problem is more serious. Both payback periods and discounted payback periods ignore the cash flows that occur towards the end of a project's life, such as the salvage value. Thus the PB is not a direct measure of profitability. The following example has a PB period of four years, which is worse than that of the previous example, but the large $15,000,000 cash inflow occurring in year five is ignored for the purposes of this metric.

Investment	Inflows				
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
-1,000,000	250,000	250,000	250,000	250,000	15,000,000

Since the payback period does not reflect the added value of a capital budgeting decision, it is usually considered the least relevant valuation approach. However, if liquidity is a vital consideration, PB periods are of major importance.

5.3. Net present value (NPV)

Net present value is the difference between the present value of cash inflows and the present value of cash outflows. NPV compares the value of a dollar today to the value of that same dollar in the future, taking inflation and returns into account. NPV analysis is sensitive to the reliability of future cash inflows that an investment or project will yield and is used in capital budgeting to assess the profitability of an investment or project.

NPV is calculated using the following formula:

$$NPV = \sum_{n=0}^{T} \frac{CF_n}{(1+r)^n}$$

Where:

n– the number of periods in the valuation model

r– the discount rate (the rate of return that could be earned on an investment in the financial markets with similar risk.); the opportunity cost of capital

CF_n = cash flow in period n

For educational purposes, NCF0 is commonly placed to the left of the sum to emphasize its role as (minus) the investment.

$$NPV = \sum_{n=1}^{T} \frac{CF_n}{(1+r)^n} - I_0$$

If the NPV of a prospective project is positive, the project should be accepted. However, if NPV is negative, the project should probably be rejected because cash flows will also be negative.

The net present value approach is the most intuitive and accurate valuation approach to capital budgeting problems. Discounting the after-tax cash flows by the weighted average cost of capital allows managers to determine whether a project will be profitable or not. And unlike the IRR method, NPVs reveal exactly *how* profitable a project will be in comparison to alternatives. The NPV rule states that all projects which have a positive net present value should be accepted while those that are negative should be rejected. If funds are limited and all positive NPV projects cannot be initiated, those with the high discounted value should be accepted.

Example: Assume Newco is deciding between two machines (Machine A and Machine B) in order to add capacity to its existing plant. Using the cash flows in the table below, let's calculate the NPV for each machine and decide which project Newco should accept. Assume Newco's cost of capital is 8.4%.

Expected after-tax cash flows machine A

Investment	Inflows					
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
-5,000	500	1,000	1,000	1,500	2,500	1000

Expected after-tax cash flows machine B

Investment	Inflows					
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
-2,000	500	1,500	1,500	1,500	1,500	1500

$$NPV_A = \frac{500}{1.084^1} + \frac{1000}{1.084^2} + \frac{1000}{1.084^3} + \frac{1500}{1.084^4} + \frac{2500}{1.084^5} + \frac{1000}{1.084^6} - 5000 = \$469$$

$$NPV_B = \frac{500}{1.084^1} + \frac{1500}{1.084^2} + \frac{1500}{1.084^3} + \frac{1500}{1.084^4} + \frac{1500}{1.084^5} + \frac{1500}{1.084^6} - 2000 = \$3929$$

Given that both machines have NPV > 0, both projects are acceptable. However, for mutually exclusive projects, the decision rule is to choose the project with the greatest NPV. Since the $NPV_B > NPV_A$, Newco should choose the project for Machine B.

Some of the major advantages of the NPV approach include the overall usefulness and easy understandability of the figure. NPV provides a direct measure of added profitability, allowing one to simultaneously compare multiple mutually exclusive projects and even though the discount rate it subject to change, a sensitivity analysis of the NPV can typically signal any overwhelming potential future concerns. Although the NPV approach is subject to fair criticisms that the value-added figure does not factor in the overall magnitude of the project, the profitability index (PI), a metric derived from discounted cash flow calculations, can easily fix this concern.

5.4. Internal rate of return (IRR)

The internal rate of return (IRR) is frequently used by corporations to compare and decide between capital projects. The internal rate of return on an investment or project is the "rate of return" that makes the net present value of all cash flows (both positive and negative) from a particular investment equal to zero. It can also be defined as the discount rate at which the present value of all future cash flow is equal to the initial investment or in other words the rate at which an investment breaks even.

For example, a corporation will evaluate an investment in a new plant versus an extension of an existing plant based on the IRR of each project. In such a case, each new capital project must produce an IRR that is higher than the company's cost of capital. Once this hurdle is surpassed, the project with the highest IRR would be the wiser investment, all other factors (including risk) being equal.

The IRR formula can be very complex depending on the timing and variances in cash flow amounts. Without a computer or financial calculator, IRR can only be computed by trial and error. One of the disadvantages of using IRR is that all cash flows are assumed to be reinvested at the same discount rate, although in the real world these rates will fluctuate, particularly with longer term projects. IRR can be useful, however, when comparing projects of equal risk, rather than as a fixed return projection.

The simplest example of computing an IRR is a mortgage with even payments. Assume an initial mortgage amount of $200,000 and monthly payments of $1,050 for 30 years. The IRR (or implied interest rate) on this loan annually is 4.8%.

Because the stream of payments is equal and spaced at even intervals, an alternative approach is to discount these payments at a 4.8% interest rate, which will produce a net present value of $200,000. Alternatively, if the payments are raised to, say $1,100, the IRR of that loan will rise to 5.2%.

The formula for IRR, using this example, is as follows

$$\sum_{n=0}^{T} \frac{CF_n}{(1+IRR)^n} = 0 \qquad \text{or} \qquad \sum_{n=1}^{T} \frac{CF_n}{(1+IRR)^n} = I_0$$

In capital budgeting, the IRR rule is as follows:

IRR > cost of capital = accept project

IRR < cost of capital = reject project

The primary advantage of implementing the internal rate of return as a decision-making tool is that it provides a benchmark figure for every project that can be assessed in reference to a company's capital structure. The IRR will usually produce the same types of decisions as net present value models, and it allows firms to compare projects on the basis of returns on invested capital.

Although IRR is easy to compute with either a financial calculator or computer software, there are some downfalls to using this metric. Similar to the PB method, the IRR does not give a true sense of the value that a project will add to a firm - it simply

provides a benchmark figure for what projects should be accepted based on the firm's cost of capital. The internal rate of return does not allow for an appropriate comparison of mutually exclusive projects; therefore managers might be able to determine that project A and project B are both beneficial to the firm, but they would not be able to decide which one is better if only one may be accepted.

Another error arising with the use of IRR analysis presents itself when the cash flow streams from a project are unconventional, meaning that there are additional cash outflows following the initial investment. Unconventional cash flows are common in capital budgeting since many projects require future capital outlays for maintenance and repairs. In such a scenario, an IRR might not exist, or there might be multiple internal rates of return. In the example below two IRRs exist - 12.7% and 787.3%.

Investment	Inflows				
Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
-1,000,000	10,000,000	-1,000,000	0	0	0

The IRR is a useful valuation measure when analyzing individual capital budgeting projects, not those which are mutually exclusive. It provides a better valuation alternative to the PB method, yet falls short on several key requirements.

5.5. Profitability index (PI)

A profitability index attempts to identify the relationship between the costs and benefits of a proposed project. The profitability index is calculated by dividing the present value of the project's future cash flows by the initial investment. A PI greater than 1.0 indicates that profitability is positive, while a PI of less than 1.0 indicates that the project will lose money. As values on the profitability index increase, so does the financial attractiveness of the proposed project.

The PI ratio is calculated as follows:

$$PI = \sum_{n=1}^{T} \frac{CF_n}{(1+r)^n} \div I_0$$

A ratio of 1.0 is logically the lowest acceptable measure for the index. Any value lower than 1.0 would indicate that the project's PV is less than the initial investment, and the project should be rejected or abandoned. The profitability index rule states that the ratio must be greater than 1.0 for the project to proceed.

For example, a project with an initial investment of $1 million and present value of future cash flows of $1.2 million would have a profitability index of 1.2. Based on the profitability index rule, the project would proceed. Essentially, the PI tells us how much value we receive per dollar invested. In this example, each dollar invested yields $1.20.

The profitability index rule is a variation of the NPV rule. In general, if NPV is positive, the profitability index would be greater than 1; if NPV is negative, the

profitability index would be below 1. Thus, calculations of PI and NPV would both lead to the same decision regarding whether to proceed with or abandon a project.

However, the profitability index differs from NPV in one important respect: being a ratio, it ignores the scale of investment and provides no indication of the size of the actual cash flows. So it is a useful tool for ranking projects because it allows you to quantify the amount of value created per unit of investment.

5.6. Discounted Payback Period (DPB)

The discounted payback period model is the capital budgeting procedure used to determine the profitability of a project. In contrast to an NPV analysis, which provides the overall value of a project, a discounted payback period gives the number of years it takes to break even from undertaking the initial expenditure. Future cash flows are considered are discounted to time "zero." This procedure is similar to a payback period; however, the payback period only measure how long it take for the initial cash outflow to be paid back, ignoring the time value of money.

Projects that have a negative net present value will not have a discounted payback period, because the initial outlay will never be fully repaid. This is in contrast to a payback period where the gross inflow of future cash flows could be greater than the initial outflow, but when the inflows are discounted, the NPV is negative.

Going back to our earlier example of Newco and the decision about which machine to purchase, let's determine the discounted payback period for Machine A and Machine B, and determine which project Newco should accept. Recall that Newco's cost of capital is 8.4%.

Discounted Cash Flows for Machine A and Machine B

Year	0	1	2	3	4	5	6
Mashine A							
Cash Flow	-5000	500	1000	1000	1500	2500	1000
Discounted Cash Flow	-5000	461	851	785	1086	1670	616
Cumulative Cash Flow	-5000	-4539	-3688	-2903	-1816	-146	469
Mashine B							
Cash Flow	-2000	500	1500	1500	1500	1500	1500
Discounted Cash Flow	-2000	461	1277	1178	1086	1002	925
Cumulative Cash Flow	-2000	-1539	-262	915	2002	3004	3929

$$PB_{Mashine\ A} = 5 + \frac{146}{616} = 5.24 \qquad PB_{Mashine\ B} = 2 + \frac{262}{1178} = 2.22$$

Machine A violates management's maximum payback period of five years and should thus be rejected. Machine B meets management's maximum payback period of five years and has the shortest payback period.

Unit 6:   Risk-Analysis

6.1. Types of Risk

Like anything, projects do have risks. There are two types of project risks associated with capital budgeting:

1. Business Risk

A company's business risk is the risk of the firm's assets when no debt is used. Business risk is the risk inherent in the company's operations. As a result, there are many factors that can affect business risk: the more volatile these factors, the riskier the company. Some of those factors are as follows:

- Sales risk - Sales risk is affected by demand for the company's product as well as the price per unit of the product.
- Input-cost risk - Input-cost risk is the volatility of the inputs into a company's product as well as the company's ability to change pricing if input costs change.

As an example, let's compare a utility company with a retail apparel company. A utility company generally has more stability in earnings. The company has les risk in its business given its stable revenue stream. However, a retail apparel company has the potential for a bit more variability in its earnings. Since the sales of a retail apparel company are driven primarily by trends in the fashion industry, the business risk of a retail apparel company is much higher. Thus, a retail apparel company would have a lower optimal debt ratio so that investors feel comfortable with the company's ability to meet its responsibilities with the capital structure in both good times and bad.

2. Financial Risk

A company's financial risk, however, takes into account a company's leverage. If a company has a high amount of leverage, the financial risk to stockholders is high - meaning if a company cannot cover its debt and enters bankruptcy, the risk to stockholders not getting satisfied monetarily is high.

Let's use the troubled airline industry as an example. The average leverage for the industry is quite high (for some airlines, over 100%) given the issues the industry has faced over the past few years. Given the high leverage of the industry, there is extreme financial risk that one or more of the airlines will face an imminent bankruptcy.

6.2. Risk-Analysis Techniques

It is important to keep in mind that when a company analyzes a potential project, it is forecasting potential not actual cash flows for a project. As we all know, forecasts are based on assumptions that may be incorrect. It is therefore important for a company to perform a sensitivity analysis on its assumptions to get a better sense of the overall risk of the project the company is about to take.

There are four risk-analysis techniques that should be known:

1. Sensitivity Analysis

Sensitivity analysis is simply the method for determining how sensitive our NPV analysis is to changes in our variable assumptions. To begin a sensitivity analysis, we must first come up with a base-case scenario. This is typically the NPV using assumptions we believe are most accurate. From there, we can change various assumptions we had initially made based on other potential assumptions. NPV is then recalculated, and the sensitivity of the NPV based on the change in assumptions is determined. Depending on our confidence in our assumptions, we can determine how potentially risky a project can be.

2. Scenario Analysis

Scenario analysis takes sensitivity analysis a step further. Rather than just looking at the sensitivity of our NPV analysis to changes in our variable assumptions, scenario analysis also looks at the probability distribution of the variables. Like sensitivity analysis, scenario analysis starts with the construction of a base case scenario. From there, other scenarios are considered, known as the "best-case scenario" and the "worst-case scenario". Probabilities are assigned to the scenarios and computed to arrive at an expected value. Given its simplicity, scenario analysis is one the most frequently used risk-analysis techniques.

3. Monte Carlo Simulation

Monte Carlo simulation is considered to be the "best" method of sensitivity analysis. It comes up with infinite calculations (expected values) given a number of constraints. Constraints are added and the system generates random variables of inputs. From there, NPV is calculated. Rather than generating just a few iterations, the simulation repeats the process numerous times. From the numerous results, the expected value is then calculated.

4. Probability analysis

The purpose of probability analysis is to eliminate the need for restricting judgement to a single optimistic, pessimistic or realistic estimation by identifying the possible range of each variable and attaching a probability of occurrence to each possible value of the variables within this range. These judgements take the form of probability distribution each possible value of each variable is associated with a number between 0 and 1, so that for each variable the sum of all these numbers (probabilities) is equal to one. This numerical description of the likelihood of an event's occurrence makes possible an objective measure of many situations that could otherwise be gauged only intuitively. Therefore, from a mathematical point of view, probability analysis consists of aggregating probabilities.

Unit 7: Bonds & Stocks

7.1. Bonds

Investment in new plant and equipment requires money - often a lot of money. Sometimes firms may be able to save enough out of previous earnings to cover the cost of investments, but often they need to raise cash from investors. In broad terms, we can think of two ways to raise new money from investors: borrow the cash or sell additional shares of common stock.

If companies need the money only for a short while, they may borrow it from a bank; if they need it to make long-term investments, they generally issue bonds, which are simply long-term loans. When companies issue bonds, they promise to make a series of fixed interest payments and then to repay the debt. As long as the company generates sufficient cash, the payments on a bond are certain. In this case bond valuation involves straightforward time-value-of-money computations. But there is some chance that even the most blue-chip company will fall on hard times and will not be able to repay its debts. Investors take this default risk into account when they price the bonds and demand a higher interest rate to compensate.

Bond is a debt investment in which an investor loans money to an entity (corporate or governmental) that borrows the funds for a defined period of time at a fixed interest rate. Bonds are used by companies, municipalities, states and countries to finance a variety of projects and activities.

Governments and corporations borrow money by selling bonds to investors. The money they collect when the bond is issued, or sold to the public, is the amount of the loan. In return, they agree to make specified payments to the bondholders, who are the lenders. When you own a bond, you generally receive a fixed interest payment each year until the bond matures. This payment is known as the coupon because most bonds used to have coupons that the investors clipped off and mailed to the bond issuer to claim the interest payment. At maturity, the debt is repaid: the borrower pays the bondholder the bond's face value (equivalently, its par value).

How do bonds work? Consider a Treasury bond as an example. Several years ago, the Treasury raised money by selling 6 percent coupon, 2015 maturity, Treasury bonds. Each bond has a face value of $1,000. Because the coupon rate is 6 percent, the government makes coupon payments of 6 percent of $1,000, or $60 each year.

When the bond matures in 2015, the government must pay the face value of the bond, $1,000, in addition to the final coupon payment.

Suppose that in 2012 you decided to buy the "6s of 2015," that is, the 6 percent coupon bonds maturing in 2015. If you planned to hold the bond until maturity, you would then have looked forward to the cash flows depicted in table below

Year	2012	2013	2014	2015
Cash Flow	- Price	$ 60	$ 60	$ 1060

The initial cash flow is negative and equal to the price you have to pay for the bond. Thereafter, the cash flows equal the annual coupon payment, until the maturity date in 2015, when you receive the face value of the bond, $1,000, in addition to the final coupon payment. How much would you be willing to pay for this stream of cash flows?

To find out, you need to look at the interest rate that investors could earn on similar securities. In 2012, Treasury bonds with 3-year maturities offered a return of about 5.6 percent. Therefore, to value the 6s of 2015, we need to discount the prospective stream of cash flows at 5.6 percent:

$$PV = \frac{60}{1.056^1} + \frac{60}{1.056^2} + \frac{1060}{1.056^3} = 1010,77$$

Bond prices are usually expressed as a percentage of their face value. Thus we can say that our 6 percent Treasury bond is worth 101.077 percent of face value, and its price would usually be quoted as 101.077.

Did you notice that the coupon payments on the bond are an annuity? In other words, the holder of our 6 percent Treasury bond receives a level stream of coupon payments of $60 a year for each of 3 years. At maturity the bondholder gets an additional payment of $1,000. Therefore, you can use the annuity formula to value the coupon payments and then add on the present value of the final payment of face value:

$$PV = PV_{Coupons} + PV_{Face\ Value} = Coupon \times \left[\frac{1-(1+r)^{-n}}{r} \right] + Face\ Value \times (1+r)^{-n} =$$

$$= \$60 \times \left[\frac{1-(1+0.056)^{-3}}{0.056r} \right] + \$1000 \times (1+0.056)^{-3} = \$161.57 + \$849.20 = \$1010.77$$

7.2. Stocks

Instead of borrowing cash to pay for its investments, a firm can sell new shares of common stock to investors. Whereas bond issues commit the firm to take a series of specified interest payments to the lenders, stock issues are more like taking on new partners. The stockholders all share in the fortunes of the firm according to the number of shares they hold.

The cash payoff to owners of common stocks comes in two forms: (1) cash dividends and (2) capital gains or losses. Usually investors expect to get some of each. Suppose that the current price of a share is P_0 , that the expected price a year from now is P_1, and that the expected dividend per share is DIV_1

The subscript on P_0 denotes time zero, which is today; the subscript on P_1 denotes time 1, which is 1 year hence. We simplify by assuming that dividends are paid only once a year and that the next dividend will come in 1 year. The rate of return that investors expect from this share over the next year is the expected dividend per share DIV_1 plus the expected increase in price P_1-P_0, all divided by the price at the start of the year P_0:

$$\text{Expected return} = r = \frac{DIV_1 + P_1 - P_0}{P_0}$$

Let us now look at how our formula works. Suppose Blue Skies stock is selling for $75 a share ($P_0$= $75). Investors expect a $3 cash dividend over the next year (DIV_1=$3). They also expect the stock to sell for $81 a year hence ($P_1$ = $81). Then the expected return to stockholders is 12 percent:

$$r = \frac{\$3 + \$81 - \$75}{\$75} = 0.12 = 12\%$$

We saw how to work out the expected return on Blue Skies stock given today's stock price and forecasts of next year's stock price and dividends. You can also explain the market value of the stock in terms of investors' forecasts of dividends and price and the expected return offered by other equally risky stocks. This is just the present value of the cash flows the stock will provide to its owner:

$$\text{Price Today} = P_0 = \frac{DIV_1 + P_1}{1 + r}$$

We have managed to explain today's stock price P_0 in terms of the dividend DIV_1 the expected stock price next year P_1. But future stock prices are not easy to forecast directly, though you may encounter individuals who claim to be able to do so. A formula that requires tomorrow's stock price to explain today's stock price is not generally helpful.

As it turns out, we can express a stock's value as the present value of all the forecast future dividends paid by the company to its shareholders without referring to the future stock price. This is the dividend discount model:

P_0 = PV of (DIV_1, DIV_2, DIV_3, … DIV_t …)

$$P_0 = \frac{DIV_1}{(1+r)^1} + \frac{DIV_2}{(1+r)^2} + ... + \frac{DIV_t}{(1+r)^t} + ...$$

The dividend discount model says that no-growth shares should sell for the present value of a constant, perpetual stream of dividends. We learned how to do that calculation when we valued perpetuities earlier. Just divide the annual cash payment by the discount rate. The discount rate is the rate of return demanded by investors in other stocks of the same risk:

$$P_0 = \frac{DIV_1}{r}$$

If dividends grow at a steady rate (g), then

$$P_0 = \frac{DIV_1}{r - g}$$

References

1. Advances in investment analysis and portfolio management / Ed. Cheng-Few Lee. - Stamford (Conn.) : JAI press, Cop. 2000

2. Bangs, David H. The cash flow control guide : Methods to understand a. control the small business' number one problem / David H. Bangs, jr. - 2. ed. - Dover (N.H.) : Upstart publ. co., Cop. 1990. - XI, 69 c.

3. Bierman H., Smidt S. The Capital Budgeting Decision: Economic Analysis of Investment Projects. 9th Ed. N.Y.: Routledge 2007

4. Boyd, G. Capital investment: a guide to making better decisions (CAI, 2013)

5. Frank K. Reilly, Keith C. Brown. Investment analysis and portfolio management

6. Osteryoung, Jerome S. Capital budgeting : Long-term asset selection / By Jerome S. Osteryoung. - 2. print. - Columbus (Ohio) : Grid, Cop. 1974. – 251

7. R.A. Brealey, S.C. Myers and F. Allen, Principles of Corporate Finance, McGraw-Hill, 9th edition, 2008.

8. Watson, D. Corporate finance- principles and practice (Financial Times Prentice Hall, 2013)

9. Экономический анализ реальных инвестиций : Учеб. : Для вузов по фин.-экон. спец. / И. В. Липсиц, В. В. Коссов. – 2-е изд., перераб. и доп. – М. : Магистр, 2007. – 380 с.

10. http://www.investopedia.com/

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